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Development of a Mathematical Model for Predicting Drying Rates of Cassava Noodles

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Abstract— This research aimed at the development of a mathematical model for predicting drying rates of cassava noodles. The model was developed using the Buckingham Pi theorem approach. The cassava noodles were dried at two levels of sample thicknesses of 0.48 and 0.72 cm; and two levels of air velocities of 1.5 and 2.5 m/s. Results from the drying experiment showed that the drying rates of cassava noodles increased with increase in air velocity and decreased with increasing sample thickness of cassava noodles. The developed model predicted very well the drying rates of cassava noodles, giving coefficient of determination (R2) values of 0.997 for both velocities of 1.5 and 2.5 m/s at sample thickness of 0.48 cm; and 0.981 and 0.963 respectively for velocities of 1.5 and 2.5 m/s of sample thickness of 0.72 cm. A two-tail test conducted using Fishers Least Significance Difference (F-LSD) approach showed no statistical difference between the predicted and experimental drying rates.

Index Terms: Buckingham Pi theorem, cassava thickness, cassava noodles, air velocity, statistical difference, drying rate.

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1 Introduction

The main purpose of drying agricultural materials is to provide longer periods of storage, minimize packaging requirements, and reduce transportation weights and costs [1]. The rates at which agricultural crops are dried determine to a large extent, the storability and shelf life of the product, the quality of the dried product, and the sprouting ability (germinability) of seeds and grains. Cassava noodle (tapioca) is a by-product of cassava which normally exists in two forms namely: long thread-like strands and rectangular shaped flakes. The cassava noodles in the form of long thread-like strands were used in this research.

Several researchers have worked on thin-layer modeling of some agricultural materials like roselle (Hibiscus Sabdariffa.L), pumpkin seed, carrot slices, cassava chips, and salted fish fillets [2, 3, 4, 5, 6]. A mathematical model for predicting the drying rate of cocoa bean in a hot air dryer was developed by [7]. However, information on developing a mathematical model for predicting the drying rate for thin-layer solar drying of cassava noodles is scarce in literature, hence the need for this research. The objectives of this research are (1) to develop a mathematical model for predicting the drying rate of cassava noodles, (2) to validate the developed model.

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2 MATERIALS AND METHODS

2.1 Experimental Procedure

The dryer used for the experiment is a forced convection integral type flat plate solar collector. It consists of a solar collector chamber and dryer cabinet both of which were integrated into one unit, fan, thermal storage unit, and air exit. The collector has an area of 0.86m2. The surface of the dryer is tilted at an angle of 50 29I to the horizontal which is the latitude of Owerri. The top of the collector is covered with plain glass while the base is lined with dark painted pebbles, which act as a thermal storage. A galvanized steel plate painted dull black was used as an absorber plate. This absorber plate overlay the thermal storage unit. A hole of diameter 15 cm at the dryer entrance was used for mounting the fan, while a hole of dimensions 8 cm x 8 cm drilled at the exit, serves as the exit for exhaust air. The dryer also has two access doors to control the operations.

The cassava noodles were dried from average initial moisture content of 202% dry basis to equilibrium moisture content of 9.8% dry basis. Two sample thicknesses of 0.48 and 0.72 cm; and two air flow velocities of 1.5 and 2.5 m/s were used for the experiment. A digital weighing balance of sensitivity 0.01 g was used to measure the mass of the samples at hourly intervals. Also a digital hygrometer was used to measure temperature and relative humidity of drying air at hourly intervals.

The moisture contents (dry basis) of the samples expressed in percentage were determined using Equation (1).

$$M(db) = \frac{m_{wp} - m_{dp}}{m_{dp}}(100\%) \quad (1)$$

Where M = moisture content of materials (g H2O/100 g dry matter)

m_{wp} = mass of wet product (g)

m_{dp} = mass of dry product (g)

db = dry basis

The drying rate of the samples were determined using Equation (2)

$$R_d = \frac{m_w}{t} = \frac{M_i - M_f}{t} \quad (2)$$

Where Mi = initial moisture content of the sample (g H2O/100 g dry matter)

M_f = final moisture content of the sample (g H2O/100 g dry matter)

m_w = mass of water removed during the drying period

t = drying time (hours)

2.2 Model Development

The drying rate model for cassava noodles was developed using dimensional analysis of Buckingham Pi theorem approach. The model parameters and their dimensions are summarized in Table (1).

Table 1: Model Parameters Affecting the Drying Rates of Cassava Noodles

Parameters	Symbols	Dimensions
Temperature	T	$M^0L^0T^0\Theta^1$
Air velocity	V	$M^0L^1T^{-1}\Theta^0$
Relative humidity	RH	$M^0L^0T^0\Theta^0$
Solar radiation intensity	I	$M^1L^0T^{-3}\Theta^0$
Slice thickness of cassava noodles	B	$M^0L^1T^0\Theta^0$
Drying time	t	$M^0L^0T^1\Theta^0$
Moisture content at a given time	M	$M^0L^0T^0\Theta^0$

The following assumptions were made in the development of the model

1. Heat transfer equations are neglected
2. Moisture diffusivity is constant throughout the drying process
3. The dimensions of the cassava noodles is constant at the same moisture content

The number of parameters that influenced the drying rate of cassava noodles (n) was 7 and the number of fundamental units (m) was 4. Therefore the number of π terms (Nπ) is represented in Equation (3).

$$N_\pi = n - m = 7 - 4 = 3\pi \; terms \quad (3)$$

There are m (4) repeating variables and these include: air velocity, solar radiation intensity, slice thickness of crop, and temperature. The drying rate (Rd) of the cassava noodles can now be expressed as Equation (4):

$$R_d = f\,\{T, V, RH, I, B, t, M\} \quad (4)$$

The π terms are now represented in Equations (5), (6), and (7).

$$\pi_1 = RH^1 * V^{a_1} I^{b_1} B^{c_1} T^{d_1} \quad (5)$$

$$\pi_2 = M^1 * V^{a_2} I^{b_2} B^{c_2} T^{d_2} \quad (6)$$

$$\pi_3 = t^1 * V^{a_3} I^{b_3} B^{c_3} T^{d_3} \quad (7)$$

The values of a, b, c, and d for the different dimensionless π groups, were obtained by algebraic calculations. These values when substituted back into Equations (5), (6), and (7) gave the dimensionless π groups as Equations (8), (9), and (10).

$$\pi_1 = RH \quad (8)$$

$$\pi_2 = M \quad (9)$$

$$\pi_3 = Vt/B \quad (10)$$

The drying rate was then expressed as a function of the π terms and expressed in Equation (11)

$$R_d = f\,\{\pi_1, \pi_2, \pi_3\} \quad (11)$$

According to [8], the dimension terms can be reduced to a manageable level either by multiplication or division by combining the π-terms. Therefore π_1 and π_2 were combined by multiplication to form π_{12} as expressed in Equation (12).

$$\pi_{12} = RH * M \quad (12)$$

Therefore the drying rate was finally expressed as Equation (13).

$$R_d = f\,\{\pi_{12}, \pi_3\} = \; f\,(RH * M * VT/B) \quad (13)$$

3 RESULTS AND DISCUSSION

3.1 Results of Drying Rates of Cassava Noodles

The results of experimental drying rates and values of the π terms for the different drying conditions are summarized in Tables (2).

Table 2: Results of Experimental Drying Rates and π terms for Different Drying Conditions

a: V = 1.5 m/s, B = 0.48 cm

M (% db)	t (hrs)	RH (dec)	Rd (%/hr)	Π_{12}	Π_3
161	0.5	0.2594	82	41.76	1.56
109	2	0.2594	46.5	28.27	6.25
55	4	0.2594	36.8	14.27	12.5
23	6	0.2594	29.8	5.97	18.75
12	8	0.2594	23.8	3.11	25
9.8	9	0.2594	21.3	2.54	28.13

b: V = 2.5 m/s, B = 0.48 cm

M (% db)	t (hrs)	RH (dec)	Rd (%/hr)	Π_{12}	Π_3
161	0.5	0.2295	98	36.95	2.6
84	2	0.2295	59	19.28	10.42
38	4	0.2295	41	8.72	20.83
15	6	0.2295	31.2	3.44	31.25
9.8	7.5	0.2295	25.6	2.25	39.06

c:: V = 1.5 m/s, B = 0.72 cm

M (% db)	t (hrs)	RH (dec)	Rd (%/hr)	Π_{12}	Π_3
179	0.5	0.281	46	50.3	1.04
132	2	0.281	35	37.09	4.17
82	4	0.281	30	23.04	8.33
46	6	0.281	26	12.93	12.5
24	8	0.281	22.3	6.74	16.67
12	10	0.281	19	3.37	20.83

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d: V = 2.5 m/s, B = 0.72 cm

M (% db)	t (hrs)	RH (dec)	Rd (%/hr)	Π_{12}	Π_3
175	0.5	0.235	54	41.13	1.74
126	2	0.235	38	29.61	6.94
68	4	0.235	33.5	15.98	13.89
45	6	0.235	26.2	10.58	20.83
14	8	0.235	23.5	3.29	27.78

The model equations were derived by plotting the experimental drying rate against one of the π terms at a time while keeping the other π term constant. Consequently, the experimental drying rates were plotted against values of π12 while keeping π3 constant. Also the experimental drying rates were plotted against values of π3 while keeping π12 constant. These plots are shown in Figus 1, 2, 3, and 4.

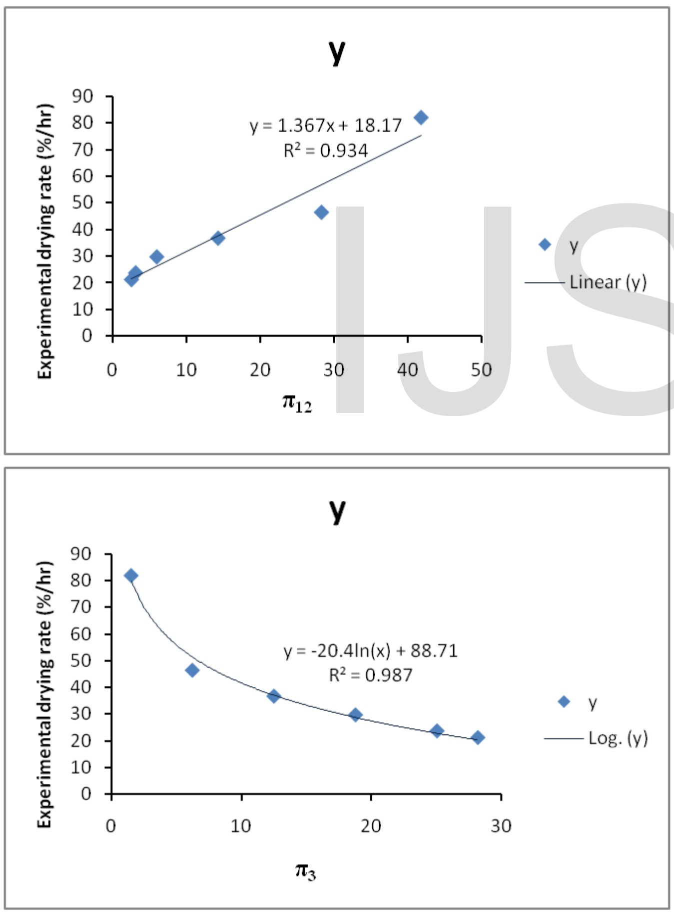


Fig 1: Experimental values of drying rate versus the π terms for velocity of 1.5 m/s and thickness of 0.48 cm.

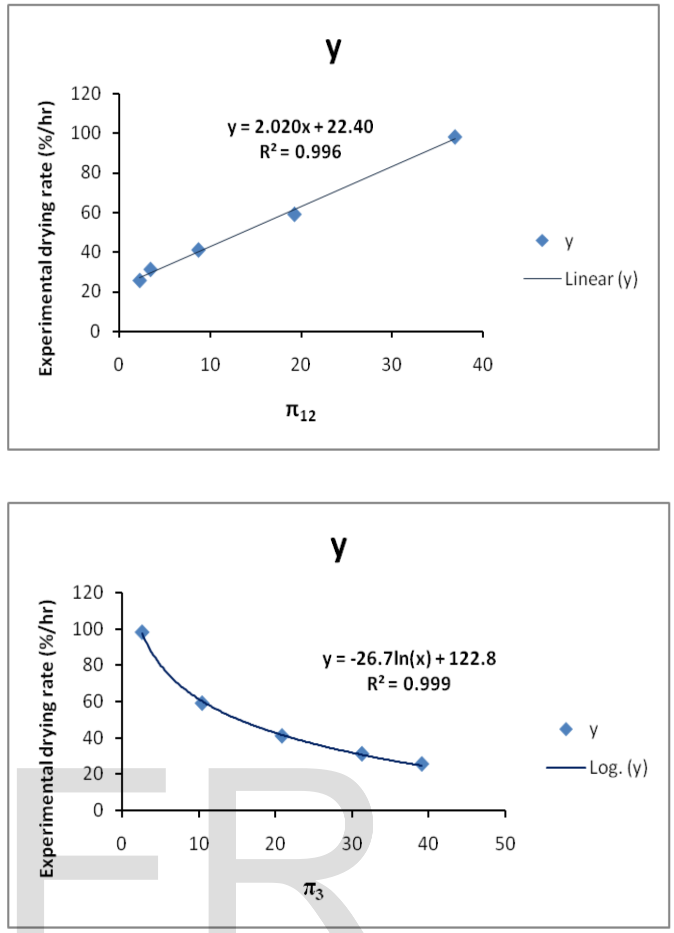
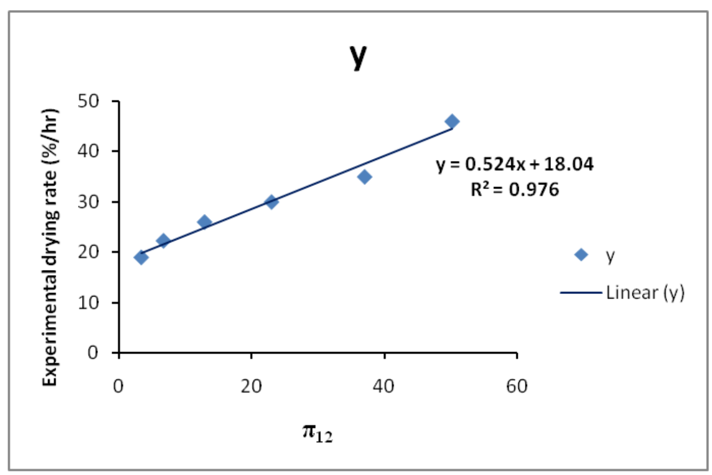


Fig 2: Experimental values of drying rate versus the π terms for velocity of 2.5 m/s and thickness of 0.48 cm.



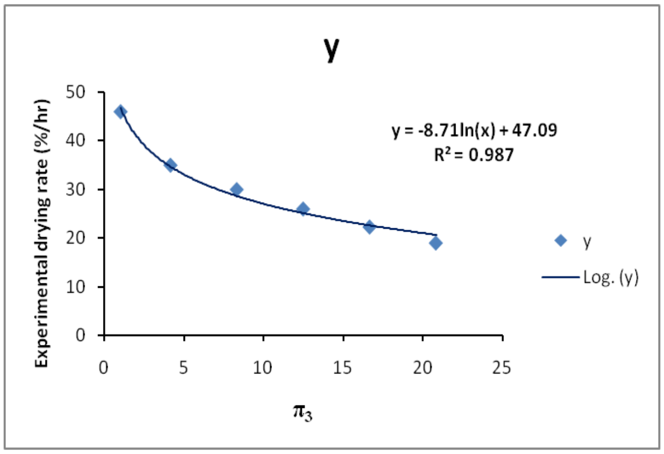
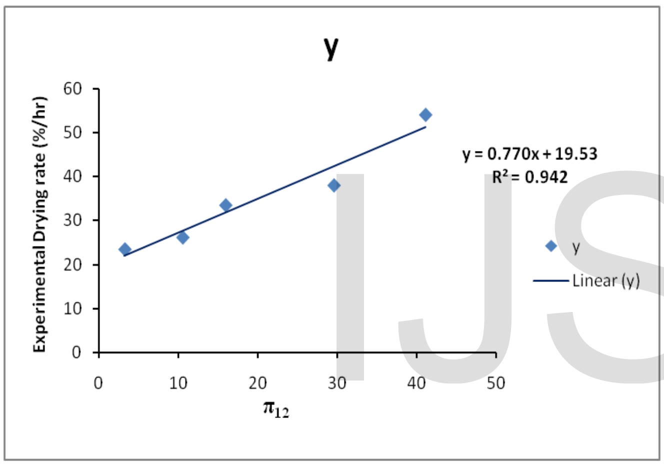


Fig 3: Experimental values of drying rate versus the π terms for velocity of 1.5 m/s and thickness of 0.72 cm.



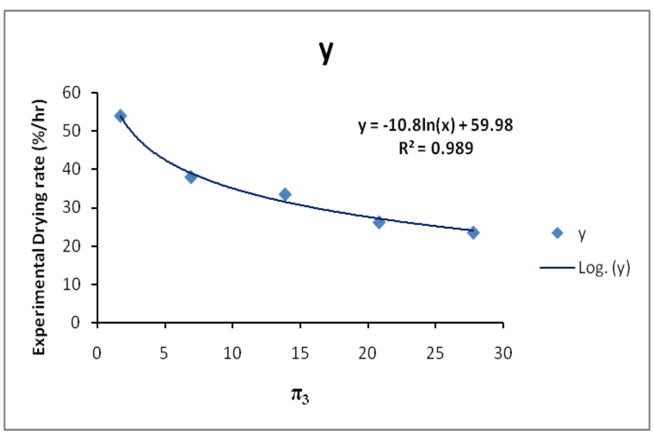


Fig 4: Experimental values of drying rate versus the π terms for velocity of 2.5 m/s and thickness of 0.72 cm.

The drying model was obtained by finding the average of the equations for π12 and π3, and substituting for the values of π12 and π3 into the various equations. The drying models for the various treatments are given in Equations (14), (15), (16), and (17).

V=1.5 m/s; B= 0.48 cm:

$$R_d = 0.684(RH * M) - 10.2\ln(Vt/B) + 53.44 \quad (14)$$

V= 2.5 m/s; B=0.48 cm

$$R_d = 1.01(RH * M) - 13.36\ln(Vt/B) + 72.6 \quad (15)$$

V=1.5 m/s; B=0.72 cm

$$R_d = 0.262(RH * M) - 4.36\ln(Vt/B) + 32.57 \quad (16)$$

V=2.5 m/s; B=0.72 cm

$$R_d = 0.385(RH * M) - 5.4\ln(Vt/B) + 39.76 \quad (17)$$

3.2 Model Validation

The developed model was validated using data obtained from solar drying of cassava noodles in a forced convection solar dryer. The validation was done at two levels of velocities (1.5 and 2.5 m/s); and two levels of sample thicknesses (0.48 and 0.72 cm); at four and five moisture content levels. The values of the predicted and experimental drying rates are given in Table (3). From the table it was evident that the predicted and experimental values are close. The two-tail test conducted using the Fisher's Least Significance Difference (F-LSD) method showed that the means of the predicted and experimental values of drying rates are not statistically different at 1% and 5% levels of probability. This conclusion was made since the calculated F-LSD value is greater than the difference in the means of predicted and experimental drying rates for all the treatments in the experiment.

Table 3: Predicted and Experimental Drying Rates for the Varying treatments of the Experiment

B=0.48 cm				B= 0.72 cm			
V=1.5m/s		V=1.5m/s		V=1.5m/s		V=2.5m/s	
R_{dp}	R_{de}	R_{dp}	R_{de}	R_{dp}	R_{de}	R_{dp}	R_{de}
66.7	62	79.99	75	41.22	41	47.33	44
44.3	41.7	49.09	48.3	32.38	32	35.88	35
31.97	33	33.92	36.2	26.62	28.8	28.6	31
24.5	26.9	26.87	27.4	23.32	24	25.6	24
				21.04	20.6	22.06	21.4

Rdp=predicted drying rate, Rde=experimental drying rate

Graphical plots of predicted drying rates versus the experimental drying rates (Figs 5 and 6) produced linear curves, with coefficients of determination (R2) ranging from 0.963 to 0.997.

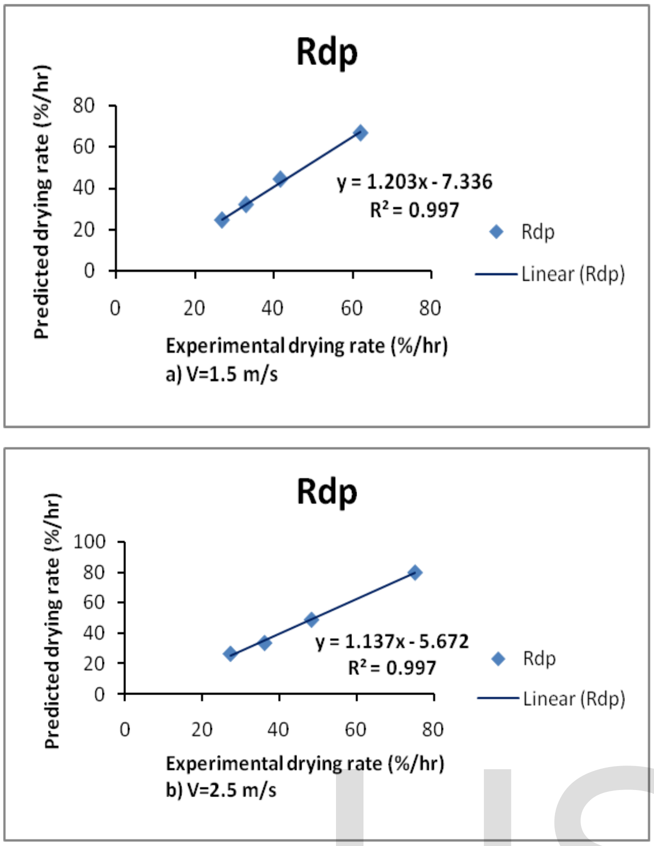
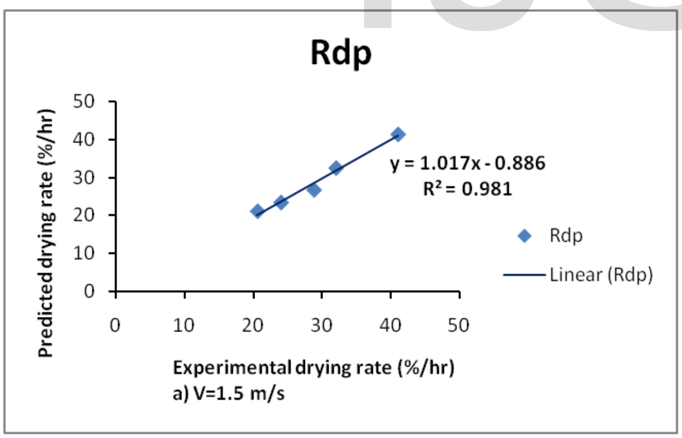


Fig 5: Relationship between Predicted and Experimental drying rates of cassava noodles dried at thickness of 0.48 cm



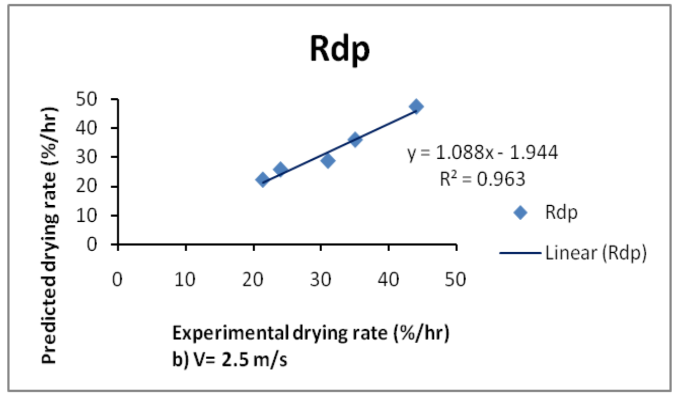


Fig 6: Relationship between Predicted and Experimental drying rates of cassava noodles dried at thickness of 0.72 cm

4 CONCLUSION

The aim of this research was to develop a mathematical model for predicting the drying rate of cassava noodles. In the course of carrying out the research, the drying model was successfully developed and validated using data obtained from solar drying of cassava noodles in a forced convection solar dryer. The experimental drying rates were closely correlated to the drying rates obtained from the developed model. This implied that the developed model predicted to a high degree of accuracy the drying rate of cassava noodles.

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